ACOUSTIC ENERGY OF A NONISOTHERMAL JET

L. M. Vyaz'menskaya UDC 532.522.2

The acoustic energy generated because of density (temperature) fluctuations and the energy due to velocity pulsations in nonisothermal jets are examined. A method is proposed to compute the noise intensity in a nonisothermal jet field.

1. To determine the acoustic power of a jet it is customary to use the expression for the noise intensity emitted by an elementary jet volume which is available, for example, in [1, 2]:

$$dN \sim \frac{\rho^2 u'^4}{\rho_0 a_0^5} \omega_{mo}^4 L^3 dW. \tag{1}$$

The difference between the acoustic intensity of an isothermal and nonisothermal jet is here taken into account by considering the density in the first case as $\rho = \rho_0$ and in the second as $\rho = \bar{\rho}(\vec{r})$. The expression (1) characterizes the acoustic energy generated because of velocity pulsations in turbulent jets. As the degree of jet heating increases, the temperature fluctuations increase and if their relative intensity in isothermal jets is slight compared with the relative intensity of the velocity pulsations, then the temperature (or density) fluctuations in nonisothermal jets are an additional source of noise formation which is not taken into account by the dependence (1).

The method of computing the acoustic radiation by turbulent jets, elucidated in [3, 4], can be used to determine the acoustic intensity of nonisothermal jets. The range of jet outflow velocities under consideration is hence bounded by subsonic or low supersonic velocities. According to [3], the pressure fluctuation at an arbitrary point outside the jet is:*

$$p'(\vec{r}_0, t) = \frac{1}{4\pi} \iiint_W \left[\frac{\partial^2}{\partial x_i \partial x_j} (\rho u_i u_j - \overline{\rho u_i u_j}) \right]_\tau \frac{dW}{|\vec{r}_0 - \vec{r}|}. \tag{2}$$

Let us consider the expression in the square bracket which represents the instantaneous value of the density and the velocity components as the sum of means and fluctuations:

$$\frac{\partial^2}{\partial x_i \partial x_j} (\rho u_i u_j - \overline{\rho u_i u_j}) = \frac{\partial^2}{\partial x_i \partial x_j} \{ \bar{\rho} (2U_i u_j' + u_i' u_j' - \overline{u_i' u_j'})$$
$$+ U_i [U_j \rho' + 2(\rho' u_j' - \overline{\rho' u_j'}) + \rho' u_i' u_j' - \overline{\rho' u_i' u_j'} \}.$$

It has been established by approximate computations that the contribution of the terms containing $\partial^2/\partial x_i \partial x_j \times [\bar{\rho}(u_i' u_j' - \overline{u_i' u_j'})]$ to the total jet acoustic intensity is substantially less than from the terms with $\partial^2/\partial x_i \partial x_j \times (2\bar{\rho} U_i u_j')$; the terms $\partial^2/\partial x_i \partial x_j [2U_i(\rho' u_j' - \overline{\rho' u_j'})]$ can be neglected as compared with $\partial^2/\partial x_i \partial x_j (U_i U_j \rho')$; and the terms containing the triple correlations are also negligible. Therefore, the pressure fluctuation outside the nonisothermal jet is

$$p'(\vec{r}_0, t) = \frac{1}{4\pi} \iiint_W \left[\frac{\partial^2}{\partial x_i \partial x_j} (2\bar{\rho} U_i u_j' + U_i U_j \rho') \right]_\tau \frac{dW}{|\vec{r}_0 - \vec{r}|}. \tag{3}$$

The pressure fluctuation outside an isothermal jet is hence obtained by putting $\bar{\rho} = \rho_0$ and $\rho' = 0$:

* Here and henceforth, the summation is from one to three in the subscript repeated twice in a monomial.

A. A. Zhdanov Leningrad State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 27, No. 4, pp. 687-693, October, 1974. Original article submitted October 18, 1973.

$$p'(\vec{r}_0, t) = \frac{\rho_0}{2\pi} \iiint\limits_W \left[\frac{\partial^2}{\partial x_i \partial x_j} (U_i u_j') \right]_\tau \frac{dW}{|\vec{r}_0 - \vec{r}|}. \tag{4}$$

Comparison shows that the influence of jet heating on its acoustic intensity is manifested in a more complicated manner than by the simple replacement of ρ_0 by $\bar{\rho}(\vec{r})$ in the expression for the acoustic intensity of an isothermal jet, as was assumed in [2], for instance. In addition to the term with $\bar{\rho}\,(\partial^2/\partial x_i \partial x_j)\,(U_i u_j')$ corresponding to the expression in (4), in which ρ_0 is replaced by $\bar{\rho}\,(\vec{r})$, differentiation with respect to the coordinates in (3) still yields terms with density fluctuations, with the derivatives of the density fluctuations with respect to the coordinates, and terms containing derivatives of the mean density.

2. Let us consider the acoustic energy determined by the term containing the density fluctuations in (3). Let us go over to cylindrical coordinates, coupled to an axisymmetric jet, and by differentiating in the sum $\partial^2/\partial x_i \partial x_j\,(\rho' U_i U_j)$, we estimate the order of smallness of the various components by taking into account the following relationships for the gasdynamic parameters [5, 6]:

$$\frac{U}{U_a} \sim 1; \quad x\frac{\partial}{\partial r}\left(\frac{U}{U_a}\right) \sim 5; \quad \frac{\partial U}{\partial x} = -\frac{\partial U}{\partial r} z; \quad \frac{\partial V}{\partial r} \sim \frac{\partial U}{\partial x}; \quad \frac{V}{U_a} \sim 0.004.$$

Let us use the relation between the time derivatives in the stream in a fixed coordinate system and in a coordinate system moving at the convection velocity $(\partial/\partial\tau)$:

$$\frac{\partial}{\partial \tau} = U_c \frac{\partial}{\partial x} + \frac{\partial}{\partial t}, \tag{5}$$

from which in conformity with [4]

$$\frac{\partial}{\partial x} \sim \frac{1}{U_c} \omega_{mo} \left(\frac{1}{\Phi} - \frac{\omega_s}{\omega_{mo}} \right); \quad \frac{\partial}{\partial r} \sim \frac{L_x}{L_r} \frac{\partial}{\partial x}.$$

On the basis of experimental results in [7], for instance, let us assume that

$$U_c \sim 0.6 U_a; \quad \omega_{mo} = \frac{1}{3} \frac{\partial U}{\partial r}; \quad \omega_s \sim 3\omega_{mo}; \quad L_x \sim 3.5 L_r.$$

The relationships presented above permit us to see that the member $U^2\,(\partial^2 \rho'/\partial x^2)$ in the sum $\partial^2/\partial x_i \partial x_j \times (\rho' U_i U_j)$ exceeds all the rest substantially. Henceforth keeping just this member, we obtain an expression for the noise intensity generated because of the density fluctuations, by the method elucidated in [3, 4]:

$$I(\vec{r}_0) = \frac{K\rho_0}{16\pi^2 a_0^3} \iiint\limits_W \frac{\bar{\rho}}{\rho_0} \left(\frac{U}{U_c} \right)^4 \frac{\overline{\rho'^2}}{\rho_0^2} \frac{\omega_{mo}^6}{\Phi^5} \left(\frac{1}{\Phi} - \frac{\omega_s}{\omega_{mo}} \right)^4 \frac{L_x^3 L_r^2}{|\vec{r}_0 - \vec{r}|^2}\, dW. \tag{6}$$

It is hence taken into account that the local speed of sound in a nonisothermal jet is a function of the coordinate $a^2 = a_0^2(\rho_0/\rho)$. The value of K is defined here by the volume of the vortex domain and the time of intrinsic lag, as for isothermal jets. The noise intensity in a direction perpendicular to the jet axis, determined by (6) with an empirical dependence for ω_{mo} taken into account, is proportional to the sixth power of the outflow velocity. The directivity of the acoustic radiation is characterized by the dependence $1/\Phi^5 \times (1/\Phi - \omega_s/\omega_{mo})^4$ and is much less definite than the directivity of isothermal jets.

The distribution of the root-mean-square density fluctuations in (6) can be found by assuming the density fluctuations in a nonisothermal jet to be due just to the temperature fluctuations. We then have from the Clapeyron equation of state

$$\frac{\rho'}{\bar{\rho}} = -\frac{T'}{\bar{T}}.$$

Using the equation of state for the mean parameters, we obtain

$$\frac{\rho'}{\rho_0} = -\frac{T'}{T_a} \frac{\bar{\rho}}{\rho_a} \frac{\bar{\rho}}{\rho_0}. \tag{7}$$

We determine the root-mean-square temperature fluctuations by using the relationship between the semi-empirical theory of free turbulence

$$\sqrt{\overline{T'^2}} = l_T \left| \frac{\partial T}{\partial r} \right|, \quad \mathrm{Pr}_t = \frac{l_u}{l_T}. \tag{8}$$

Here l_u, l_T are the mixing paths corresponding to the momentum and heat transfer. Theoretical and experimental investigations permit assuming the turbulent Prandtl number Pr_t to be constant in the jet cross-section and equal to 0.5 (see [5]). On the basis of experimental investigations of the distributions of the root-mean-square velocity fluctuations in jets, presented in [6], say, we can put $l_u = 0.03x$. According to [6], the excess temperature distribution in the initial section of the jet is given by the dependence

$$\frac{T_a - T}{T_a - T_0} = 1 - \eta, \quad \eta = -3.7z + 0.534; \tag{9}$$

and by

$$\frac{T - T_0}{T_m - T_0} = 1 - \xi^{3/2}, \quad \xi = 3.7z. \tag{10}$$

in the main section. Substituting of (9), (10) into (8), we obtain a dependence characterizing the distribution of the root-mean-square temperature fluctuations in the initial jet section

$$\frac{\sqrt{\overline{T'^2}}}{T_a} = 0.22\left(1 - \frac{1}{\theta}\right) \tag{11}$$

and in the main section

$$\frac{\sqrt{\overline{T'^2}}}{T_m} = \frac{1}{3}\sqrt{\xi}\left(1 - \frac{T_0}{T_m}\right). \tag{12}$$

Because of the assumptions made about the constancy of Pr_t and l_u/x in the jet cross-section and the independence of $x(\partial T/\partial r)$ from the coordinates in the initial section, the root-mean-square temperature fluctuations are independent of the coordinates. Their relative intensity increases as the degree of jet heating grows, and reaches 22% for large θ.

3. In order to determine the acoustic energy due to the velocity fluctuations in a nonisothermal jet, let us consider the sum $\partial^2/\partial x_i \partial x_j (2\bar{\rho} U_i u'_j)$ in (3). Here direct differentiation yields a large quantity of components for which an analysis of their contribution to the total acoustic radiation of the jet is made difficult. This expression is easily evaluated approximately if it is assumed that the density in the jet is just a function of the coordinates. Since the acoustic energy generated because of the density fluctuations has already been determined by (6), neglecting the density fluctuations for the approximate determination of the acoustic energy generated by the turbulent velocity pulsations is admissible. Two conditions, obtained from the continuity equation

$$\frac{\partial(\bar{\rho} U_i)}{\partial x_i} = 0, \quad \frac{\partial}{\partial x_i}(\bar{\rho} u'_i) = 0,$$

are a result of such an assumption, and using them we have

$$\frac{\partial^2}{\partial x_i \partial x_j}(\bar{\rho} U_i u'_j) = \bar{\rho}\frac{\partial U_i}{\partial x_j}\frac{\partial u'_j}{\partial x_i} + \frac{1}{\bar{\rho}} U_i u'_j \frac{\partial \bar{\rho}}{\partial x_i}\frac{\partial \bar{\rho}}{\partial x_j} - U_i u'_j \frac{\partial^2 \bar{\rho}}{\partial x_i \partial x_j}.$$

Going over to cylindrical coordinates here and using the following identity transformation

$$\frac{1}{\bar{\rho}}\frac{\partial \bar{\rho}}{\partial x_i}\frac{\partial \bar{\rho}}{\partial x_j} - \frac{\partial^2 \bar{\rho}}{\partial x_i \partial x_j} = -\bar{\rho}\frac{\partial^2 (\ln \bar{\rho})}{\partial x_i \partial x_j},$$

we find

$$\frac{\partial^2}{\partial x_i \partial x_j}(\bar{\rho} U_i u'_j) = \bar{\rho}\frac{\partial U}{\partial r}\frac{\partial v'}{\partial x} - \bar{\rho}\left[U v' \frac{\partial^2}{\partial x \partial r}(\ln \bar{\rho}) + V v' \frac{\partial^2}{\partial r^2}(\ln \bar{\rho}) + U u' \frac{\partial^2}{\partial x^2}(\ln \bar{\rho})\right].$$

The method in [3, 4] permits obtaining an expression for the noise intensity generated because of the velocity pulsations in nonisothermal jets:

$$I(\vec{r}_0) = \frac{K\rho_0}{4\pi^2 a_0^3} \iiint\limits_W \left(\frac{\bar{\rho}}{\rho_0}\right)^3 \frac{\omega_{mo}^2}{\Phi^5} \frac{L_r^2 L_x^3}{|\vec{r}_0 - \vec{r}|^2} \times [(B + b_1)\overline{v'^2} + b_2\overline{u'^2} + b_3\overline{u'v'}]\, dW. \quad (13)$$

Here

$$B = \left[\frac{\partial U}{\partial r} \frac{\omega_{mo}}{U_c}\left(\frac{1}{\Phi} - \frac{w_s}{w_{mo}}\right)\right]^2,$$

$$b_1 = -\left[U \frac{\partial^2}{\partial x \partial r}(\ln\bar{\rho}) + V \frac{\partial^2}{\partial r^2}(\ln\bar{\rho})\right]^2,$$

$$b_2 = -\left[U \frac{\partial^2}{\partial x^2}(\ln\bar{\rho})\right]^2,$$

$$b_3 = -2\left[U \frac{\partial^2}{\partial x \partial r}(\ln\bar{\rho}) + V \frac{\partial^2}{\partial r^2}(\ln\bar{\rho})\right]\left[U \frac{\partial^2}{\partial x^2}(\ln\bar{\rho})\right].$$

The derivatives of $\ln\bar{\rho}$ in the expressions for b_1, b_2, b_3 are evaluated by using the equation of state and the temperature distribution in a jet (9), (10). Let us estimate the ratios b_1/B, b_2/B, b_3/B in the range of the highest values of the root-mean-square velocity pulsations, i.e., for z close to zero. An estimate using the dependences for the gasdynamic parameters and the turbulence characteristics presented in Sec. 2 shows that $b_1/B < 0.01$, $b_2 \sim 0$, $b_3 \sim 0$ for $\theta \le 6$. Omitting terms containing b_1, b_2, b_3 in (13), we find that the part of the acoustic energy generated because of the velocity fluctuations is determined by the same expression as for the acoustic energy of an isothermal jet (see [4]) in which only the factor $(\bar{\rho}/\rho_0)^3$ appears under the integral sign. The additional power of $\bar{\rho}/\rho_0$ in (13), as compared with the second power of this ratio used in [1, 2], is due to taking account of the dependence of the speed of sound in the jet on the coordinates $(a_0/\bar{a})^2 = \bar{\rho}/\rho_0$.

The factor $(\bar{\rho}/\rho_0)^3$ indicates some diminution in the acoustic intensity as the degree of jet heating grows due to the velocity fluctuations. At the same time, the acoustic intensity generated because of the temperature fluctuations increases as the degree of heating grows according to (6), (11), (12). A comparison between the integrands in (6) and (13) shows that the acoustic energy generated by an elementary jet volume because of the temperature fluctuations is commensurate, and exceeds the acoustic energy generated by the velocity pulsations, for sufficiently large θ. The analysis made permits the conclusion that the noise-formation mechanism associated with the temperature fluctuations is predominant in the process of noise production by hot turbulent jets.

By using the results obtained above, the following expression can be proposed to compute the noise intensity outside a nonisothermal jet

$$I(\vec{r}_0) = \frac{K\rho_0}{4\pi^2 a_0^3} \iiint\limits_W A\left(\frac{\bar{\rho}}{\rho_0}\right)^3 \frac{1}{\Phi^5}\left(\frac{1}{\Phi} - \frac{\omega_s}{\omega_{mc}}\right)^2 \frac{L_x^3 L_r^2}{|\vec{r}_0 - \vec{r}|^2}\, dW,$$

$$A = \overline{v'^2}\, \omega_{mo}^4 \left(\frac{\partial}{\partial r} \frac{U}{U_a}\right)^2 \left(\frac{U_a}{U_c}\right)^2 + \frac{1}{4} \frac{\overline{T'^2}}{T_a^2} \left(\frac{\bar{\rho}}{\rho_a}\right)^2 \omega_{mo}^6 \left(\frac{U}{U_c}\right)^4 \left(\frac{1}{\Phi} - \frac{\omega_s}{\omega_{mo}}\right)^2. \quad (14)$$

The value of K found for isothermal jets was 0.05-0.08. It can be expected that K will be of the same order of magnitude for nonisothermal jets. The lack of systematic experimental results about the noise of nonisothermal jets does not yet permit refinement of the value of K.

NOTATION

$\vec{r}$, radius vector; x_i, x_j, its projections on the Cartesian coordinate axes; x, r, φ, cylindrical coordinates coupled to an axisymmetric jet; $z = r - 1/x$, in the initial section of the jet; $z = r/x$ in the main section; t, time; $\tau = t - |\vec{r}_0 - \vec{r}|/a_0$; u_i, u_j, the velocity components in projections on Cartesian coordinate axes; u, v, axial and radial velocity components; U_c, vortex convection velocity (the upper case denotes the mean values of the velocity, and the lower case denotes instantaneous values); p, pressure; ρ, density; T, temperature; T_m, temperature on the jet axis; $\theta = T_a/T_0$, degree of jet heating; a, speed of sound; I, noise intensity; N, acoustic intensity; W, volume occupied by the jet; ω_{mo}, ω_s, characteristic circular frequencies in the coordinate systems moving at the convection velocity and fixed, respectively;

$$\Phi = \left[\left(1-\frac{U_c}{a}\cos\alpha\right)^2+\left(\frac{\omega_{mo}L}{a_0}\right)^2\right]^{1/2};\ \cos\alpha=\frac{x_0-x}{|\vec{r}_0-\vec{r}|};\ L=\sqrt{\frac{1}{2}(L_x^2+L_r^2)};$$

L_x, L_r, longitudinal and radial integrated turbulence scales. Subscripts: 0, parameters in the unperturbed medium; *a*, parameters at the nozzle exit; primes denote pulsating components. The upper bar denotes the average.

LITERATURE CITED

1. M. J. Lighthill, AIAA Journal, No. 7 (1963).
2. A. G. Munin, Industrial Aerodynamics [in Russian], No. 23, 200 (1962).
3. L. M. Vyaz'menskaya, Inzh.-Fizich. Zh., 20, No. 4, 711 (1971).
4. L. M. Vyaz'menskaya, Vestnik Leningrad University, No. 1, 88 (1973).
5. A. S. Ginevskii, Theory of Turbulent Jets and Wakes [in Russian], Moscow (1969).
6. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Fizmatgiz (1960).
7. P. O. A. L. Davies, M. J. Fisher and M. J. Barrat, J. Fluid Mech., 15, 337 (1963).